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TECHNICAL NOTE 2946

A SMALL PIRANI GAGE FOR MEASUREMENTS
OF NONSTEADY LOW PRESSURES

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SUMMARY

The precise measurement of low absolute pressures such as are found in supersonic wind tunnels presents a growing problem, especially where these pressures are changing. A precise, stable, rapid-response Pirani gage is presented as a solution of this problem.

A description is given of a small Pirani gage (0.0005-cubic-inch internal volume) made of surgical grain-of-wheat lamps, and the operating equipment for the recording of 12 channels of low pressures in the range 0.1 to 10 mm Hg abs. Techniques of calibration and use of this apparatus are described and measurements are shown to contain errors not exceeding ± 2 percent of reading, lags not exceeding 1 second, and calibration shifts of 2 percent per year. These small Pirani gages appear to be well-suited for static-pressure measurements in low-pressure supersonic tunnels and may be useful for other applications as well.

A theoretical analysis of a small Pirani gage is given and may be used as a guide in the design, application, and evaluation of such a gage.

INTRODUCTION

With the introduction of low-density wind tunnels operating at static pressures from 0.1 to 10 mm Hg abs, there has developed a need for the measurement of these pressures under changing conditions. Instruments having adequate sensitivity for the measurement of pressures in this range (for example, inclined-tube manometers, McLeod gages, diaphragms, and electrical gages of the ionization, thermocouple, and Pirani types) have not provided precise measurements except at the cost of greatly increased lag in response due to large gage volumes and required long connecting tubing. The need for a reliable instrument with low lag is therefore evident. The basic problem in the design of such an instrument is the construction of a gage of small internal volume; in addition, a small external gage size is desirable so that many gages can be mounted close to the pressure orifices with a minimum of connecting tubing.

Of the number of physical phenomena which have been adapted for the measurement of low pressures, the principle of the Pirani gage appeared to be best-suited to serve as the basis for a gage which would be small and yet reliable. The Pirani gage is a thermal-electric instrument consisting simply of a wire filament heated electrically in the gas being measured. Gas pressure is measured as a function of the gage-filament cooling resulting from the thermal conduction of the surrounding gas.

For most gases at ordinary pressures and temperatures, the heat conductivity is independent of pressure and is a function of the absolute temperature. However, as described in reference 1, at low pressures where the spatial dimensions of an apparatus are of the order of the molecular mean free path of the gas, the rate of heat conduction between surfaces by the gas becomes a function of gas pressure as well as absolute temperature. This fact forms the basis for the pressure sensitivity of the Pirani type gage. The gas composition also affects the rate of heat conduction through the gas, so that it is necessary to calibrate and operate such a gage with gas compositions of closely similar molecular properties.

A small Pirani gage has been designed by the Instrument Research Division of the Langley Aeronautical Laboratory. This gage is made of surgical grain-of-wheat lamps which are extremely small and have a filament of a small diameter suitable for use in a Pirani gage. In the operation of these gages, the excessive error, instability, and lag which usually characterize Pirani gages have been reduced to an acceptable level.

In the present paper, the design and performance of these small Pirani gages are described and a general theoretical analysis is presented so as to aid in the construction of more or less similar devices for applications where the particular characteristics of such gages appear to be propitious.

I. CHARACTERISTICS AND PERFORMANCE OF A SMALL

PIRANI GAGE MADE OF SURGICAL LAMPS

Apparatus

Gages.— Small Pirani gages have been constructed consisting of a matched pair of surgical grain-of-wheat lamps mounted in close proximity within a metal block in order to insure the same temperature environment for the two lamps. One lamp is open to the pressure being measured, while the other, sealed at atmospheric pressure, provides partial temperature

compensation for the pressure-sensitive lamp. (The preparation of the gages is described in detail in the appendix.) The surgical grain-of-wheat lamps are each only $11/32$ inch long with 0.082-inch outside diameter. As a result, these gages may be mounted close to the pressure points being measured, with a minimum of connecting tubing, and may be incorporated in tunnel stings or within models. One gage case has been built for mounting 12 pairs of lamps in a block 1.25 inches long and 0.80 inch in diameter. Individual gages have been mounted in duralumin blocks $5/8 \times 1/4 \times 3/16$ inch. Figures 1 and 2 show the small Pirani gages mounted in gage cases with O-ring seals which make it possible to assemble and disassemble low-pressure connections in a matter of seconds. The gage case shown in these figures is relatively large, as it was designed for tunnel wall installations where case size was not critical.

The internal volume of a surgical grain-of-wheat lamp is only 0.0005 cubic inch, so that pneumatic and thermal lags attributable to this volume of air within the lamp are very small. The power dissipation from these lamp filaments is held to a small value (maximum, 25 milliwatts) so that the interior walls of these lamp bulbs are maintained at the temperature of the gage block. The temperature rise of each filament above the temperature of the lamp-bulb wall is determined by the filament current and by the efficiency of the gas molecules in conducting heat from the filament to the bulb wall. At low pressures, where the mean free paths of the gas molecules are no longer negligible compared with the gage filament diameter, then the filament temperature rise, and hence resistance, is a measurable function of the pressure of the gas within the bulb.

Circuit.— The filament resistance is measured in a constant-current Wheatstone bridge. (See fig. 3.) By having the sealed bulb in an arm of the bridge adjacent to the arm containing the pressure-sensitive bulb, partial temperature compensation is obtained. The sealed and pressure-sensitive bulbs used together in the Wheatstone bridge result in a galvanometer deflection recorded on film which is a function of the pressure of the gas within the gage and to a lesser extent of the temperature of the gage walls.

The bridge is designed so that the unbalance current is a function of the difference between the resistances of the pressure and temperature bulbs and is not measurably affected by the individual magnitudes of these resistances.

A program device operates to switch each bridge momentarily, in sequence, from a connection where the gage is used to measure pressure, to a circuit configuration where half of the gage is used as a resistance thermometer. In this way, the temperature of the gage wall is measured and used to calculate the corrections for pressure measurements made at a gage temperature different from that of the calibration.

A four-wire cable, one wire to each lamp lead, connects each gage to the rest of the Wheatstone bridge. Two of the leads, one from each lamp, are connected together at the gage. The arrangement of the electrical connections, as illustrated in figure 3, results in the cancellation of lead resistance changes with ambient-temperature changes, for both the pressure-measurement and temperature-measurement circuits.

The recorder galvanometer elements have a natural frequency of 8.5 cps, and a sensitivity of about 140 microamperes for a full-scale deflection of 2 inches. These elements are damped from critical to 66 percent of critical, depending on the element and on the range of the instrument.

Range selection is accomplished by means of a network across the bridge unbalance; three ranges are provided, giving sensitivities in the ratios 1:2:4. The 12-channel circuit (fig. 4) is made up of three sections which can be range-switched independently of each other, so that groups of four channels switch range together. Each channel is provided with a jack that can be used to transfer the indications from the recorder to a meter, so that any pressure point can be observed during a test.

Power supply.- A precise, stable current supply is required with this apparatus because this instrument is from three to twenty times more sensitive to current changes than to pressure changes. For example, at 0.3 mm Hg abs, deviations of 0.05 percent in the bridge current would cause 0.5-percent errors in pressure measurement. Since each bridge operates at a constant current of about 78.4 milliamperes, a 12-channel unit requires a total of about 1 ampere of regulated current, highly stable over a period of time at least equal to the test duration. Tests were made to determine the adaptability of standard aircraft batteries to the problem. Accurate determination was especially desired of the characteristics of the initial transients known to exist in wet-cell operation. Type AN3150 aircraft batteries under a load of 1 ampere were found to have a starting transient for the first 5 to 20 minutes of discharge, after which the terminal-voltage decay stabilized to about 0.004 percent per minute. Wet-cell batteries were found to be far superior to electronically regulated supplies in simplicity, cost, stability, freedom from ripple, and dependability. Accordingly, the power supply for this instrument makes use of a battery-supplied high-voltage bus with individual swamping resistors to provide constant currents to each channel. Six aircraft batteries, of the type AN3150 are used in series with a voltage-setting rheostat to supply a 144-volt bus voltage with the accuracy and stability required. The bus-voltage rheostat is constructed of thermocouple-type selector switches and provides bus-voltage increments of 0.05 volt.

The resistance of each individual channel is initially adjusted so that, with a bus voltage of exactly 144 volts, each channel current is of the correct value as indicated by the single-point potentiometer. During pressure measurements, the single-point potentiometer can be successively connected into each channel to check the operation of each channel and to indicate when the bus voltage requires adjustment.

Charging equipment and monitoring meters are built into the 12-channel cabinet rack (fig. 5), but the batteries themselves are installed in a room provided for such equipment, where they are not a safety hazard and where fumes from the batteries cannot cause corrosion in other measuring apparatus.

Calibration equipment.- The small surgical-lamp Pirani gages have been calibrated against a specially constructed inclined-tube butyl-phthalate manometer. The calibration apparatus consists of the inclined-tube manometer, a reference vacuum system, and a calibration vacuum system provided with O-ring receptacles into which the gages are plugged. The leak rate of this apparatus is less than 0.02 micron per second, and the vapor content is held to a negligible value by use of a cold trap refrigerated with a carbon-dioxide snow and alcohol mixture. Changes in manometer sensitivity and zero with temperature are corrected for, and lags in the manometer (up to 300 seconds) are allowed for. Emphasis has been placed on careful vacuum technique which is required in order to obtain accurate calibrations at the pressures covered by the small Pirani gage.

The inclined-tube manometer has a full-scale pressure of 20 mm Hg abs and a slope of 5:1. The glass tubing is maintained exceptionally clean in order to insure mobility of the meniscus. A differential pressure calibration of this manometer has been obtained at atmospheric pressure, and this differential pressure calibration is taken to be valid at low absolute pressures. The manometer "zero" deflection has been found to be the same for the case where the manometer is shorted at atmospheric pressure and for the case where the manometer is not shorted but negligible absolute pressures are applied to both sides. This fact indicates that errors due to leaks or outgassing are not measurable.

The precision of this manometer is limited by (a) a scale-reading probable random error of 0.2 millimeter in length which results in a probable error in pressure measurement of 0.003 mm Hg abs and (b) systematic errors not exceeding 0.5 percent of reading. (Probable and systematic errors are discussed in ref. 2.)

The calibration apparatus described has been used to obtain calibrations of the small Pirani gages over a range of pressure from 0.1 to 10 mm Hg abs.

Processing of Data

A record from a 12-channel instrument is shown in figure 6. The channel number of each trace can be determined from the temperature deflection because a programming device switches the channels from pressure measurement to momentary temperature measurement in numerical order with a pause between channel 12 and the beginning of a new cycle.

Information is read from the film records by determining the film trace deflection, that is, the distance (in inches) between the galvanometer film trace and the appropriate reference line on the film. The deflections read for any one channel (that is, trace) are:

- p pressure-trace deflection
- t temperature-trace deflection
- p₀ reference value of p obtained from an atmospheric-pressure record (on the same range and channel) obtained directly before or after a test

The original calibration graph (fig. 7) is prepared by plotting many values of pressure obtained from the calibration equipment against deflection D where

$$D = p - p_0$$

This graph is ordinarily used throughout the life of the gage. When the gages are used to make measurements at a gage-case temperature other than that of the original calibration, a computation for the deflection D is made according to the formula

$$D = (p - p_0) [1 + n(t - t_0)]$$

where the value of the temperature coefficient n is 0.6 for the apparatus under discussion. The value of t₀ is the value of t found during the original calibration, provided that the calibration has not shifted. If the calibration has shifted, as indicated by a few check calibration points, a new value of t₀ should be calculated so as to make the check points fall on the original calibration graph. This technique corrects for shifts in gage, circuit, or galvanometer characteristics, if these occur. The calculation is

$$t_0 = t - \frac{1}{n} \left(\frac{D}{p - p_0} - 1 \right)$$

where P , p , t , and p_0 are measured and D is obtained from the original calibration graph. The value of t_0 has been found to be relatively stable, and check calibrations have not been required more often than every 60 days.

Occasional checking of the value of t_0 has held measurement errors due to shifts in instrument characteristics to a small fraction of 1 percent compared with a shift in instrument characteristics of about 2 percent per year. In the sample calibration graph shown in figure 7, the value of t_0 was the same for the original calibration and for the two check calibrations shown.

Performance

The range of pressure measured by the small Pirani gage made of surgical lamps is 0.1 to 10 mm Hg abs. Operation of this instrument and processing of the records are routine and do not require engineering personnel time, and unusual precautions are not required to protect the gages from damage due to pressure, temperature, or accidental misoperation. The gage calibration has been found to be unaltered after operation at any pressure from high vacuum to atmospheric, or after an abrupt pressure change from high vacuum to atmospheric, or vice versa; as may occur in low-density apparatus. Repeated calibrations of the apparatus used have indicated that the over-all precision is within 2 percent of indication except above 4 mm Hg abs where the accuracy is limited by the amount of trace deflection available. In a year of use, the instrument characteristics have been found to shift about 2 percent, but the resulting errors introduced by this shift are held to a fraction of 1 percent by occasional check calibrations (see section entitled "Processing of Data").

Measurements made with Pirani gages are limited in precision because of errors resulting from

- (a) Changes in filament accommodation coefficient
- (b) Reduced pressure sensitivity at higher pressures
- (c) Lag
- (d) Changes in gage current
- (e) Changes in gas composition
- (f) Changes in gage-case temperature

The effects of these factors on the characteristics of the small surgical-lamp Pirani gages are described in the sections that follow.

Filament accommodation coefficient.- As described in the section of part II entitled "Accommodation Coefficient," a change in the accommodation coefficient of a Pirani gage filament will result in errors in pressure indication. A stable accommodation coefficient can be obtained by avoiding contaminating gases and excessive filament temperature. The small surgical-lamp Pirani gages have been operated with dry air or nitrogen and at a low filament temperature rise (from 40° C at atmospheric pressure to 350° C maximum temperature rise at 0.1 mm Hg abs), and the characteristics of these gages have been found to be stable.

Pressure-sensitivity characteristics.- Except at the lower pressures measured with the small Pirani gage, the deflection is approximately inversely proportional to the pressure being measured, so that a reasonably linear calibration graph is obtained on logarithmic or reciprocal paper. The fact that the calibration graphs are approximately linear is advantageous, because more data points would be required to determine curved calibration lines accurately. For the given dimensions of the small Pirani gage, an excessively high filament current would be required to extend the linearity to the lower pressures (see section of part II entitled "Gage Relations"). With the gage current in use, 78.4 milliamperes, there is no danger of gage-filament burnout (see fig. 8) or calibration shift resulting from operating these gages at any pressure from atmospheric to the lowest obtainable.

Useful trace deflections are limited to those for which 0.005 inch is a small part of the total deflection. This limitation is a result of the fact that for normal film-reading techniques the probable error in film reading is about 0.005 inch.

The average deflection characteristics are:

Range	Pressure, mm Hg abs, for -		
	2-inch (full-scale) trace deflection	0.5-inch trace deflection	0.2-inch trace deflection
Low	0.1	1.0	2.5
Medium	.5	2.0	5.0
High	1.0	4.0	10.0

Thus pressures are measured with this instrument over a pressure range of 40:1, from 0.1 to 4 mm Hg abs, with trace-reading probable errors no greater than ± 1 percent. Pressures up to 10 mm Hg abs are

measured (extending the pressure range to 100:1) with resulting increased film-reading errors up to ± 2.5 percent probable error at a pressure of 10 mm Hg abs. Large overlapping of the ranges is provided so that small trace deflections can be avoided except at the higher values of pressure.

Lag.- By itself, the gage has a lag error less than 1 percent of the instantaneous pressure in response to steady rates of pressure change as large as 10 percent of the instantaneous pressure per second. One second after an abrupt pressure change, the gage by itself has a lag error not greater than 1 percent of the final pressure.

Representative values of lag for the small surgical-lamp Pirani gage instrument are given in the following table. The lag time $t_{0.99}$ is the time taken for the instrument to respond to within 99 percent of the total change. The pressure lags were measured with the Pirani gage within a bell jar and therefore are free of connecting-tubing lag.

Step function	$t_{0.99}$, sec
Pressure change of 1.85 to 2.6 mm Hg abs	0.26
Pressure change of 2.7 to 15 mm Hg abs	.31
Gage current off to on, at a pressure of 2.7 mm Hg abs	1.2
Step function applied to recorder galvanometer alone	.14

Where the gages are used with connecting tubing, the instrument thermal lag and gage volume are so small that the response lag is ordinarily that of the tubing alone, which can be calculated.

Gage current.- As described in the section entitled "Power supply," a precise power supply is used with the small surgical-lamp Pirani gages, providing gage currents stable within 0.02 percent during a test duration. Thus even though the Pirani gage is from three to twenty times as sensitive to current changes as it is to pressure changes, the error introduced from this source has been held to less than 0.5 percent.

Gas composition.- This instrument can be used with a wide variety of gases. Three gases that result in virtually the same calibration are in use, namely, nitrogen, dry air, and room air (that is, air with a water content no greater than 2 percent by weight). The thermal conductivities of these gases are very nearly the same, and the use of any combination of these gases has been found to cause changes in sensitivity no greater than 0.5 percent from the calibration obtained with dry air.

The presence of a larger concentration of water vapor, however, would result in measurable systematic errors in the Pirani gage pressure measurements. At low pressures, water vapor may be introduced from outgassing metal or liquid surfaces so that a cold trap or some other means is required to remove the water vapor.

These gages have been used with short tubing leading to a tunnel test section where the flow of dry nitrogen prevented the accumulation of water vapor resulting from outgassing. With this arrangement, errors due to water vapor were not large enough to detect.

Gage-case temperature.- The gages are partially temperature-compensated in their construction and a further temperature correction is calculated in the processing of the records (see section entitled "Processing of Data"). A systematic temperature error exists in the final corrected deflection, but this error is less than 1 percent per 10° F difference between the gage-case temperature during the calibration check and that during the test. These gages are useful for making pressure measurements over a range of gage-case temperatures from 40° to 120° F; however, exposure to case temperatures in the range 0° to 200° F does not change the calibration of the gages.

II. THEORETICAL CHARACTERISTICS OF A SMALL PIRANI GAGE

A theoretical derivation is presented for the characteristics of a Pirani gage which is small enough dimensionally so that heat conduction through the leads and through the gas are the only measurable sources of heat power dissipation. The derivation is carried out for a gage consisting of two matched lamps, one open to the gas pressure to be measured and the other sealed at atmospheric pressure in order to temperature-compensate the first lamp. Each individual lamp is considered to consist of a wire concentrically placed in a cylindrical bulb. The derived equations provide a design guide in the selection of gage dimensions and operating current and are in a form useful for predicting the calibration curves to be expected for a constant-current gage and the effects of gas composition and of deviations in gage-case temperature and gage current. The theory has been applied to the small Pirani gages made from surgical grain-of-wheat lamps. These lamps have U-shaped filaments rather than concentrically placed ones; nevertheless, the measured characteristics have been found to agree with the theory.

Symbols

a	accommodation coefficient
c	heat-conduction coefficient, watt cm K ⁻¹
g	heat-conduction coefficient, watt cm ⁻¹ K ⁻¹
h	electrical power coefficient, watt cm ⁻¹
k	Boltzmann constant, erg K ⁻¹
l	length of wire filament, cm
m	mass of molecule, gram
q	electric power coefficient, watt cm ⁻¹ K ⁻¹
r	radius, cm
x	length along wire filament, cm
A	constant
B	constant
C _A	constant
C _B	constant
C _V	specific heat at constant volume per molecule, erg K ⁻¹
H	heat power generated electrically in filament, watts
H _C	heat power conducted along length of filament, watts
H _G	heat power dissipated from filament surface through surrounding gas, watts
I	electric current, amp
J	thermal conductivity of wire filament, watt cm ⁻¹ K ⁻¹
K	thermal conductivity of gas, watt cm ⁻¹ K ⁻¹
P	pressure, dyne cm ⁻² (microbars)

Q	constant
R	electrical resistance of filament, ohms
T	absolute temperature, °K
z	dimensionless coefficient
α_2	temperature coefficient of electrical resistance, the ratio: change of resistance in filament wire due to a change of temperature of 1° K divided by its resistance at the temperature T_2 , °K ⁻¹
β	dimensionless ratio, C_v/k
λ	molecular mean free path, cm
λ_F	molecular mean free path of an unconfined gas, cm
ρ_2	electrical coefficient of resistivity of filament wire at temperature T_2 , ohm cm
σ	diameter of gas molecule, cm
θ	dimensionless coefficient

Subscripts:

1	wire surface
2	bulb inner wall, or temperature of bulb inner wall
λ	conditions 1 mean free path distant from filament surface
o	open bulb
s	sealed bulb

Sources of Heat Power Dissipation in a Small Pirani Gage Lamp

The possible sources of heat power dissipation from a heated wire in a gas are: radiation, free convection, forced convection, conduction through the gas, and conduction through the wire terminals. Radiation losses from a fine heated wire are relatively small for temperatures below red heat. If such a heated wire is operated in a sufficiently

small volume, free convection currents are not set up (even at atmospheric pressure) because of the gas film around the wire; whereas forced convection of heat by the mass flow of gas is limited by the smallness of the reservoir for the gas to flow in or out of. As a result, it is possible to have a Pirani gage for which only the conduction of heat through the gas and through the wire terminals must be considered. This is the case for a small Pirani gage made of surgical grain-of-wheat lamps, where all sources of heat loss but these last two are less than a fraction of a percent of the total heat loss.

Heat generated electrically in filament.- The temperature T_1 at any point on the electrically heated wire filament will vary along the length of the filament, being highest at the midpoint. At the ends of the filament, at the supports, T_1 will be equal to the temperature T_2 of the bulb inner wall. At any point along the length of the filament (see fig. 9 and ref. 3), the joulean heat per differential length of filament is given by:

$$\frac{dH}{dx} = \frac{I^2 \phi_2}{\pi r_1^2} \left[1 + \alpha_2 (T_1 - T_2) \right] = h + q (T_1 - T_2) \quad (1)$$

where

$$h = \frac{I^2 \phi_2}{\pi r_1^2}$$

and

$$q = \frac{I^2 \phi_2 \alpha_2}{\pi r_1^2}$$

The value of ϕ_2 depends on the reference temperature T_2 but the value of $\phi_2 \alpha_2$ is very nearly constant, so that, for any one gage, at one value of current, the parameter q is virtually constant.

Heat conduction through filament.- The heat power conducted along the length of the filament at any point is given by (see fig. 9 and ref. 3):

$$H_c = \pi r_1^2 J \left(- \frac{dT}{dx} \right) = -c \frac{dT}{dx}$$

where

$$c = \pi r_1^2 J$$

so that

$$\frac{dH_c}{dx} = -c \frac{d^2T}{dx^2} \quad (2)$$

Heat conduction through gas from filament.- The mechanism of heat conduction from a small wire concentrically placed in a cylinder has been described in reference 4. There are two regions of heat flow that must be considered (see fig. 10):

Region 1: The region in which gaseous conduction exists whose outer boundary is the cylinder wall and whose inner boundary is 1 mean free path from the wire surface

Region 2: The region in which free molecular travel exists which is between region 1 and the wire surface

For a low power dissipation from the wire, the temperature drop between a point 1 mean free path distant from the wire surface and the cylinder inner surface is very small, so that very closely:

$$K_\lambda = K_2$$

$$P_\lambda = P_2$$

Then for a differential length of the wire, through region 1 (ref. 4):

$$\frac{dH_g}{dx} = \frac{2\pi K_2 (T_\lambda - T_2)}{\log_e \left(\frac{r_2}{r_1 + \lambda} \right)} \quad (3)$$

whereas through region 2 (ref. 4)

$$\frac{dH_g}{dx} = (\beta + 0.5) \left(\frac{2\pi k}{mT_2} \right)^{0.5} r_1 a P_2 (T_1 - T_\lambda) 10^{-7} \quad (4)$$

Combining these last two relations yields

$$\frac{dH_g}{dx} = g(T_1 - T_2) \quad (5)$$

where

$$\frac{1}{g} = \frac{\log_e \left(\frac{r_2}{r_1} \right)}{2\pi K_2} - \frac{\log_e \left(1 + \frac{\lambda}{r_1} \right)}{2\pi K_2} + \frac{\left(\frac{mT_2}{2\pi k} \right)^{0.5} 10^7}{(\beta + 0.5) r_1 a P_2} \quad (6)$$

From kinetic theory (ref. 1, pp. 25 and 113), the molecular mean free path of an unconfined gas λ_F is given by

$$\lambda_F = \frac{kT}{\pi \sigma^2 P_2 \sqrt{2}} \quad (7)$$

For values of the mean free path λ_F less than r_2 , $\lambda = \lambda_F$; while as λ_F approaches r_2 , the last relation fails and λ approaches a maximum value equal to r_2 . Thus for $\lambda_F < r_2$ (a condition which includes the range of most Pirani gages), equation (6) becomes

$$\frac{1}{g} = \frac{\log_e \left(\frac{r_2}{r_1} \right)}{2\pi K_2} - \frac{\log_e \left(1 + \frac{kT_2}{\pi \sigma^2 r_1 P_2 \sqrt{2}} \right)}{2\pi K_2} + \frac{\left(\frac{mT_2}{2\pi k} \right)^{0.5} 10^7}{(\beta + 0.5) r_1 a P_2} \quad (8)$$

If $\lambda_F \geq r_2$, equation (6) becomes

$$\frac{1}{g} = \frac{\left(\frac{mT_2}{2\pi k}\right)^{0.5} 10^7}{(\beta + 0.5)r_1 a P_2} \quad (9)$$

Kinetic theory does not provide an accurate relation for thermal conductivity, but experimentally determined values can be represented closely by Sutherland's equation, or more simply, for air near 300° K (ref. 5) by the relation:

$$K = K_{300} \left(\frac{T_2}{300}\right)^{0.8} \quad (10)$$

Accommodation Coefficient

In free molecular heat conduction between a gas and a surface, molecules strike the surface, are momentarily adsorbed, and are then emitted from the surface. The molecules generally do not come into thermal equilibrium with the surface before being emitted, so that the heat power conduction is only a fraction of that which would occur if complete thermal equilibrium were reached by each molecule before being emitted. This fraction is Langmuir's accommodation coefficient a .

Experimentally determined values of accommodation coefficient a have been found (refs. 1, 4, and 6) throughout the possible range between 0 to 1. The value of a has been shown to depend on the wire material, the composition of the surrounding gas, absolute temperature, excess of the wire temperature over the gas temperature, and on the nature of the surface of the wire. Surfaces with a film of adsorbed gases have generally been found to have values of a considerably larger than that for a free surface. If the surface condition of the wire can be maintained, then the value of a for a particular wire and gas will be a function of the excess of the wire temperature over the gas temperature, and to a small extent, of the absolute temperature. In a Pirani gage, these last two parameters are already among those that determine the instrument deflection and so are accounted for in the calibration. The surface condition of the wire, however, is a potential source of erratic changes in accommodation coefficient, with resulting calibration shifts or drifts in a Pirani gage. A stable surface condition of the wire can be obtained by avoiding contaminating gases and elevated temperatures (see section of part I entitled "Performance").

Gage-Lamp Relations

The combined effect of heat generated electrically in the lamp filament and heat losses through the filament and through the gas may now be found. In any differential length of the electrically heated filament, the joulean heat power equals the heat power loss due to conduction along the filament and to conduction from the filament surface to the gas; that is, (see fig. 9)

$$\frac{dH}{dx} = \frac{dH_g}{dx} + \frac{dH_c}{dx} \quad (11)$$

Combining equation (11) with equations (1), (2), and (5) gives:

$$h + q(T_1 - T_2) = g(T_1 - T_2) - c \frac{d^2 T_1}{dx^2}$$

or

$$\frac{d^2 T_1}{dx^2} + \frac{q - g}{c} T_1 = \frac{q - g}{c} T_2 - \frac{h}{c}$$

Replacing the constants with symbols gives:

$$\frac{d^2 T_1}{dx^2} + AT_1 = B$$

The solution of the last equation is in the form:

$$T_1 = Q_1 e^{x\sqrt{-A}} + Q_2 e^{-x\sqrt{-A}} + \frac{B}{A}$$

The limits are:

$$\frac{dT_1}{dx} = 0 \quad \left(x = \frac{l}{2}\right)$$

$$T_1 = T_2 \quad (x = 0)$$

Solving for T_1 yields

$$T_1 - T_2 = \frac{h}{q - g} \left[\frac{e^{x\sqrt{-A}} + e^{(l-x)\sqrt{-A}}}{1 + e^{l\sqrt{-A}}} - 1 \right] \quad (12)$$

Equation (12) is an expression of the temperature rise at any point on the filament. Combining equations (12) and (1) gives

$$\frac{dH}{dx} = h + \frac{hq}{q - g} \left[\frac{e^{x\sqrt{-A}} + e^{(l-x)\sqrt{-A}}}{1 + e^{l\sqrt{-A}}} - 1 \right]$$

or, over the length of the filament,

$$H = \int_0^l \left[h - \frac{hq}{q - g} + \frac{hq}{q - g} \frac{e^{x\sqrt{-A}} + e^{(l-x)\sqrt{-A}}}{1 + e^{l\sqrt{-A}}} \right] dx$$

Therefore,

$$H = hl \left\{ 1 + \frac{q}{g - q} \left[1 - \frac{\tanh \sqrt{\frac{l^2(g - q)}{4c}}}{\sqrt{\frac{l^2(g - q)}{4c}}} \right] \right\}$$

If

$$z = \frac{l^2(g - q)}{4c}$$

and

$$\theta = \frac{1 - \frac{\tanh\sqrt{z}}{\sqrt{z}}}{z} \equiv \frac{1 - \frac{\tan\sqrt{-z}}{\sqrt{-z}}}{z}$$

then

$$H = hl \left(1 + \frac{l^2 q}{4c} \theta \right)$$

or

$$I^2 R = I^2 R_2 \left(1 + \frac{l^2 q}{4c} \theta \right)$$

so that the resistance rise of the gage-lamp filament is found to be:

$$R - R_2 = R_2 \frac{l^2 q}{4c} \theta \quad (13)$$

Gage Relations

The derivation has so far been for a single lamp. In a gage, two lamps are used together, one open and the other sealed. The two lamp filaments are matched, so that the unheated resistance R_2 is the same for both lamps. Then with the two lamps operated at the same filament current:

$$R_O - R_S = R_2 \frac{l^2 q}{4c} (\theta_O - \theta_S) \quad (14)$$

The function θ can be simplified by a close approximation obtained as follows. Values of z for a small Pirani gage, such as one made of surgical grain-of-wheat lamps, will ordinarily lie between 16

and -2.4. For this range of values of z , a graphical solution (ref. 2) of θ yields, with errors less than 3 percent,

$$\theta = \frac{0.84}{z + \left(\frac{\pi}{2}\right)^2} = \frac{0.84 \frac{4c}{l^2}}{g + \frac{\pi^2 c}{l^2} - q}$$

Applying this approximation to equation (14) yields

$$R_O - R_S = C_A \left[(g_O + C_B)^{-1} - (g_S + C_B)^{-1} \right] \quad (15)$$

where

$$C_A = 0.84 R_2 q = 0.84 R_2 \frac{I^2 \rho_2 \alpha_2}{\pi r_1^2}$$

and

$$C_B = \frac{\pi^2 c}{l^2} - q = \frac{\pi^3 r_1^2}{l^2} J - \frac{I^2 \rho_2 \alpha_2}{\pi r_1^2}$$

A special case exists where $C_B = 0$, for then:

$$R_O - R_S = C_A \left(\frac{1}{g_O} - \frac{1}{g_S} \right) \quad (16)$$

Values of g_O , for an open lamp, and g_S , for a matching sealed lamp, are obtained from equation (8). The reference pressure in the sealed lamp may be either high vacuum or atmospheric. An atmospheric-pressure reference is preferable, as it permits an instrument zero adjustment at atmospheric pressure. The recorder deflection is made proportional to $R_O - R_S$. For the case of equation (16), that is, $C_B = 0$, at any one gage-case temperature T_2 the value of $R_O - R_S$ is linearly

proportional to the reciprocal of the measured pressure P_2 . Where $C_B \neq 0$, the gage characteristics are given by equation (15) and are not linear at the lower pressures. The extent of the nonlinearity depends on the magnitude of C_B . The values of C_A and C_B depend on only the gage-filament properties and the gage current, as can be seen by the relations following equation (15). Thus, the values of C_A and C_B can be adjusted by proper selection of the gage filaments and filament current.

The values of J and $\rho_2 \alpha_2$ in the derivations are assumed to be constant, whereas actually they vary slightly with filament temperature T_1 . The slight temperature dependence of $\rho_2 \alpha_2$ is due to α_2 alone, since ρ_2 is independent of T_1 , and $\rho_2 \alpha_2$ is independent of T_2 . The values of J and $\rho_2 \alpha_2$ have the greatest effect in the derived equations at the higher filament temperatures, so that the best values to use for J and α_2 are those corresponding to the maximum filament temperature of the gage under consideration.

Calculations of Theoretical Characteristics of a Small Pirani Gage

Calculations have been carried out by means of equations (8) and (15) for a small Pirani gage made of surgical grain-of-wheat lamps used to measure the pressure of dry air at 300°K . An accommodation coefficient $a = 1$ was assumed, and the following parameter values were used:

Values for dry air (refs. 1 and 3):

β near 300°K	2.5
k , ergs K^{-1}	1.38×10^{-16}
m , grams	48.11×10^{-24}
σ , cm	3.72×10^{-8}
K_2 at 300°K , watt $\text{cm}^{-1}\text{K}^{-1}$	0.263×10^{-3}

Measured values for average surgical grain-of-wheat lamp:

r_1 , cm	0.000907
r_2 , cm	0.0660
l , cm	0.648
ρ_2 at 300°K , ohm cm	7.18×10^{-6}
α_2 for $T_2 = 300^\circ \text{K}$ and $T_1 = 650^\circ \text{K}$, $^\circ\text{K}^{-1}$	0.0039
J for $T_1 = 650^\circ \text{K}$, watt $\text{cm}^{-1}\text{K}^{-1}$	1.58

The tungsten filaments of the surgical grain-of-wheat lamps are of a small diameter so that values of ρ , α , and J are substantially different from values given in the literature for larger-diameter tungsten wire. Values of ρ and α have been determined from measurements of the physical dimensions and resistance-temperature characteristics of the filaments of the surgical grain-of-wheat lamps.

The value of the filament thermal conductivity J has been calculated by using experimentally determined values of the parameters in equations (8) and (15), this being the only method available. Data for calculating J were taken at 0.1 mm Hg abs and a filament current of 78.4 milliamperes, conditions where the heat loss by gaseous conduction is a small fraction of the heat loss through the leads. Under these conditions, it was felt that errors due to accommodation coefficient and filament location would be minimized. It is realized that this procedure for determining J is approximate and results in the theoretical characteristics coinciding with calibration data at 0.1 mm Hg abs and 78.4 milliamperes.

There are three points about the calculations that are not exact: the method of obtaining J , the approximation $a = 1$, and the fact that the filaments of the surgical grain-of-wheat lamps are U-shaped rather than straight. However, the correctness of the form of the derivation is indicated by the fact that the curves calculated from equations (8) and (15) show good agreement throughout the pressure range of the gage with the curve determined experimentally for the small Pirani gages made of surgical lamps, as shown in figure 11.

CONCLUDING REMARKS

The small Pirani gage, made of surgical lamps, has been described, together with associated circuitry, calibration, application, and methods used to overcome the basic limitations of this type of instrument. The operating characteristics have been measured to be: lags less than 1 second, range from 0.1 to 10 mm Hg abs, and errors less than 2 percent of reading except at the higher pressures (above 4 mm Hg abs) where precision is limited by film-reading probable errors of up to 2.5 percent at 10 mm Hg abs.

Equations have been derived which provide design information for the selection of Pirani gage dimensions and operating current and indicate the necessary degree of control or correction for the effects of gas composition and deviations in gage temperature and current. Theoretical characteristics of the small surgical-lamp Pirani gage have been calculated and have agreed with measured characteristics.

It is felt that the description of performance in part I and the theory of part II serve as a guide in the construction and use of similar gages for the measurement of low pressures with low lag of response.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., February 25, 1953.

APPENDIX

PREPARATION OF GAGES

A drawing of the surgical grain-of-wheat lamps used in the construction of the small Pirani gages is shown in figure 1. These lamps consist of a single-loop tungsten filament (0.00071 inch in diameter), two platinum connectors which pass through the glass bulb seal, and two copper leads (0.0145 inch in diameter) 2 inches long. These lamps have been made by the General Electric Co. Each lamp is first annealed by operation at 1.5 volts for 2 minutes; this relieves the stresses in the wire and stabilizes the internal structure and surface of the wire. The lamp resistance with the filament at room temperature is next accurately determined for each lamp by measuring its resistance at a current which produces a negligible temperature rise in the tungsten filament (for example, 1 milliampere). These resistances are all measured at a room temperature of 72° F or else are converted to this temperature to obtain, for each lamp, a value of R_{72} , the lamp resistance with the filament at 72° F. Values of R_{72} have been found to range from 1.58 to 3.34 ohms because of differences in the construction of the individual lamps. Lamps with values of R_{72} from 1.70 to 2.30 ohms have been found to produce satisfactory gages, and such lamps are selected for further processing. The value of R_{72} is remeasured at each stage during the processing, and any lamp showing erratic changes in this value is rejected.

Each lamp is opened by filing down the tip of the bulb opposite from the leads; care is taken so that a minimum of powdered glass is drawn into the lamp when the seal is broken. A matched pair of lamps is selected for each gage, where the resistance mismatch between the two lamps is not greater than 0.01 ohm at 1 milliampere or at 80 milliamperes. The hole in one lamp from each pair is next resealed at atmospheric pressure either by fusing the glass bulb tip with a fine neutral oxyacetylene flame, or else with Armstrong Products Co. adhesive A-2 prepared with their activator "A." The sealed lamp becomes the temperature-compensating element in the gage.

The paired lamps are mounted in a metal case as close together as possible in order to reduce the effect of any temperature gradients. Many gage case configurations are possible (see figs. 1 and 2). The lamps are sealed into the appropriate mounting holes (0.086 inch in diameter) in the metal gage case. Seals are made with Armstrong's A-2 adhesive. After the adhesive is air-dried for several days, a good vacuum seal is produced.

When electrical leads have been added, the gage is connected to the electrical circuit. (See fig. 3.) Each gage is evacuated to about 0.1 mm Hg abs and maintained at this low pressure for several days in order to liberate adsorbed gases and vapors. Finally, the gage is operated at this low pressure at the correct gage current (about 78.4 milliamperes) in order to stabilize and degass the gage filament. Then the gage is ready for calibration and use.

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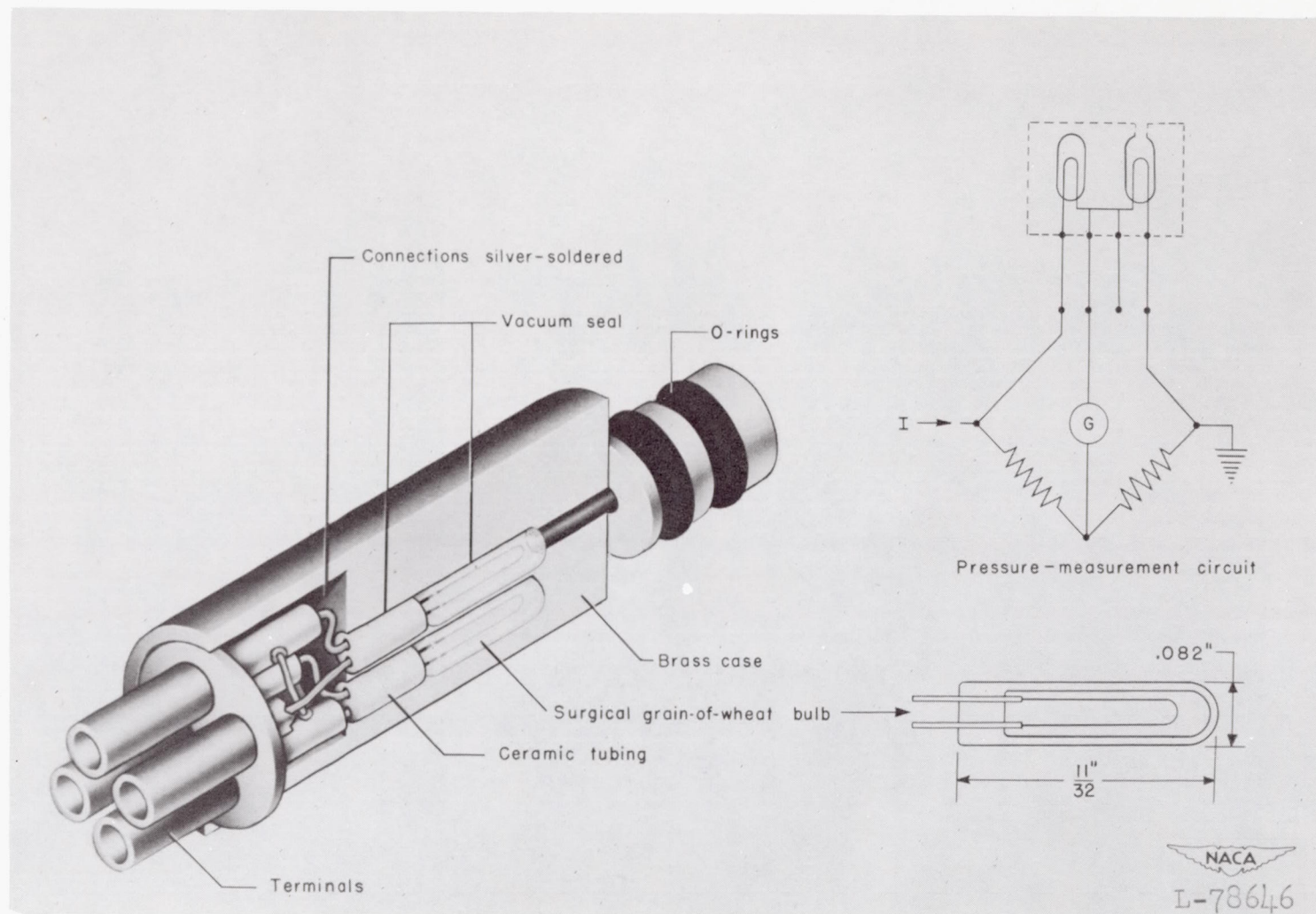
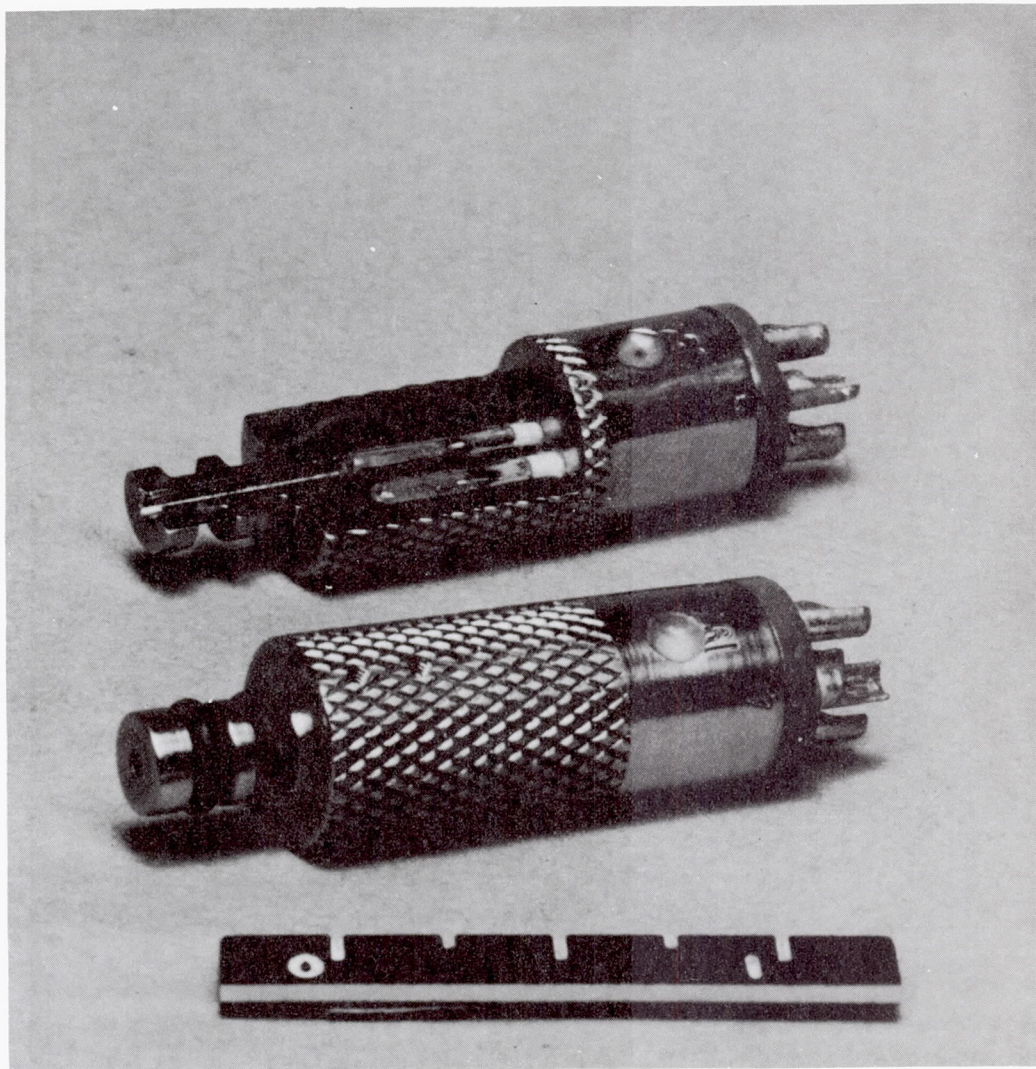


Figure 1.- Cutaway view of small Pirani gage made of surgical lamps mounted in O-ring-type gage case.

The NACA logo, consisting of the letters "NACA" inside a stylized wing shape.

L-78519

Figure 2.- Photograph of small Pirani gage made of surgical lamps mounted in O-ring-type gage case; cutaway case shown in background.

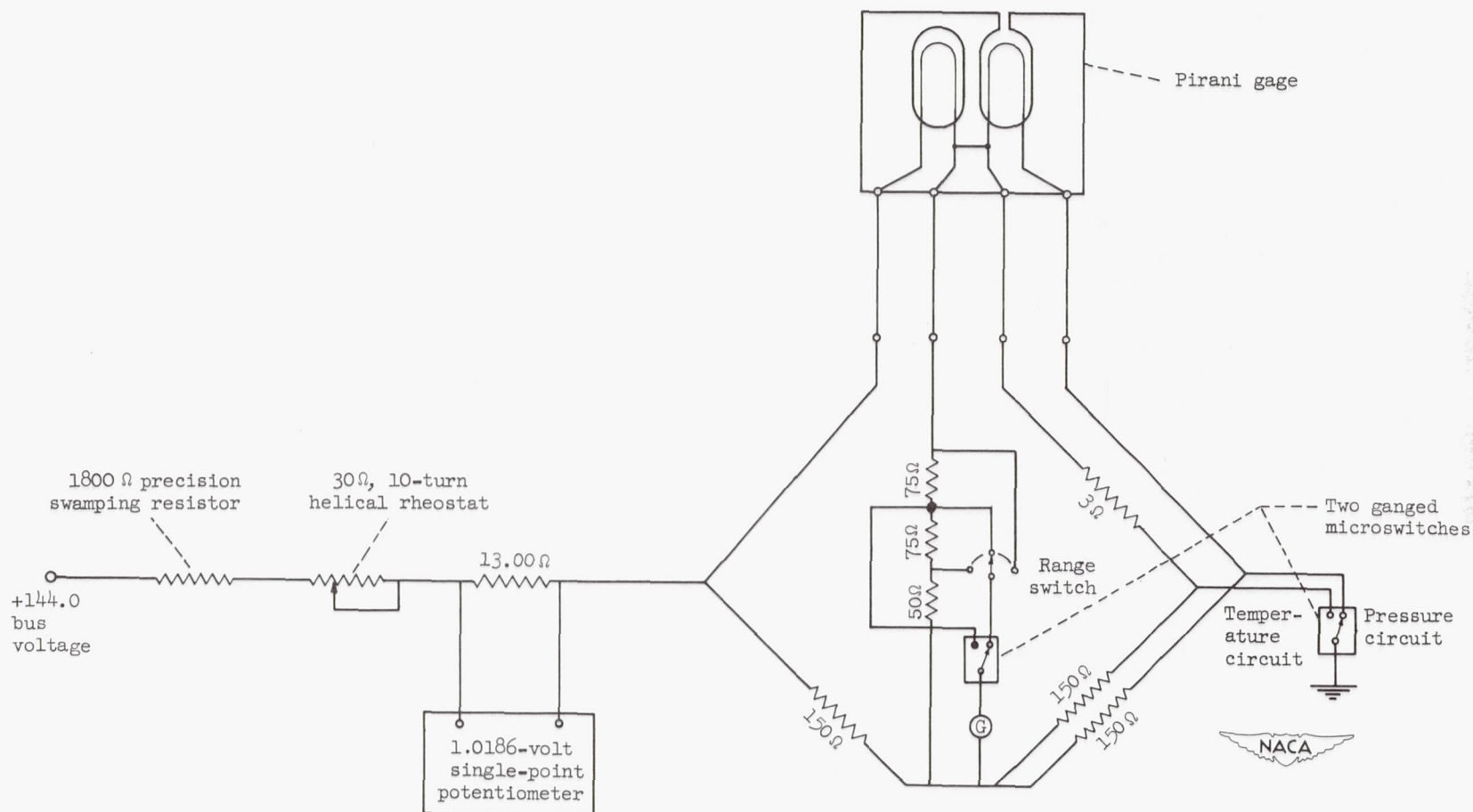


Figure 3.- Circuit diagram of one channel of Pirani gage instrument.

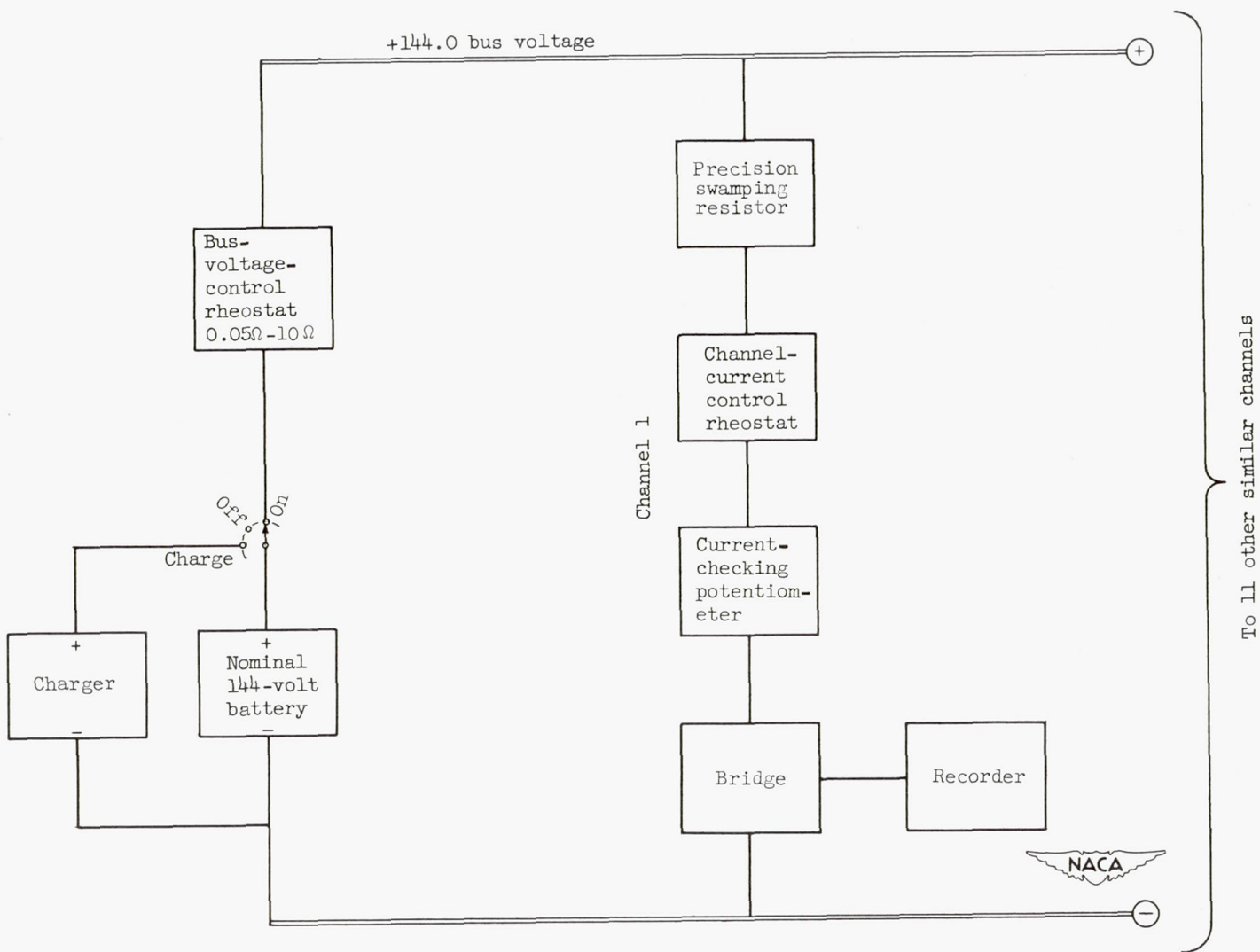


Figure 4.- Block diagram of surgical-lamp Pirani gage 12-channel low-pressure recorder.

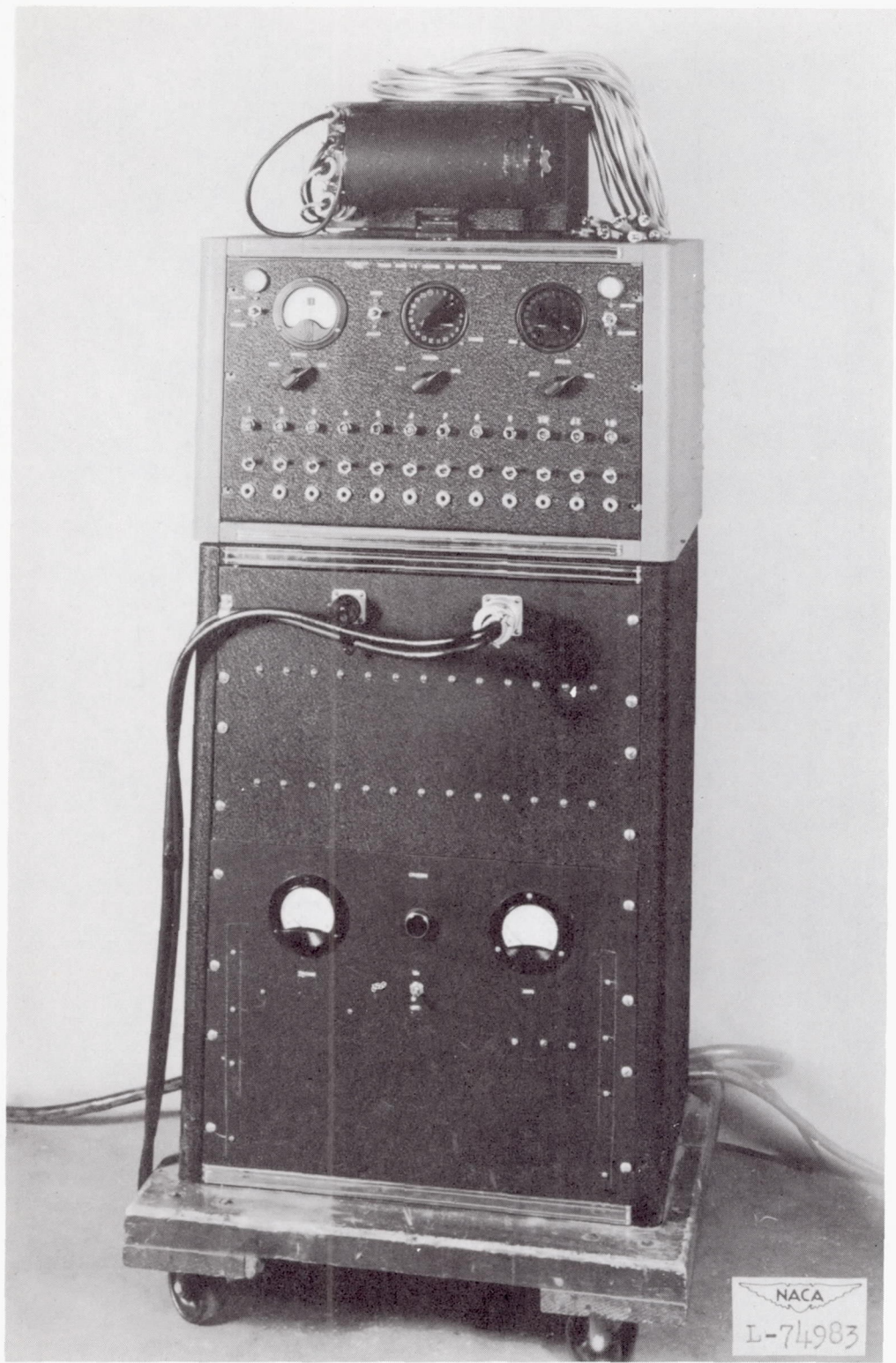


Figure 5.- Photograph of surgical-lamp Pirani gage 12-channel low-pressure recorder.

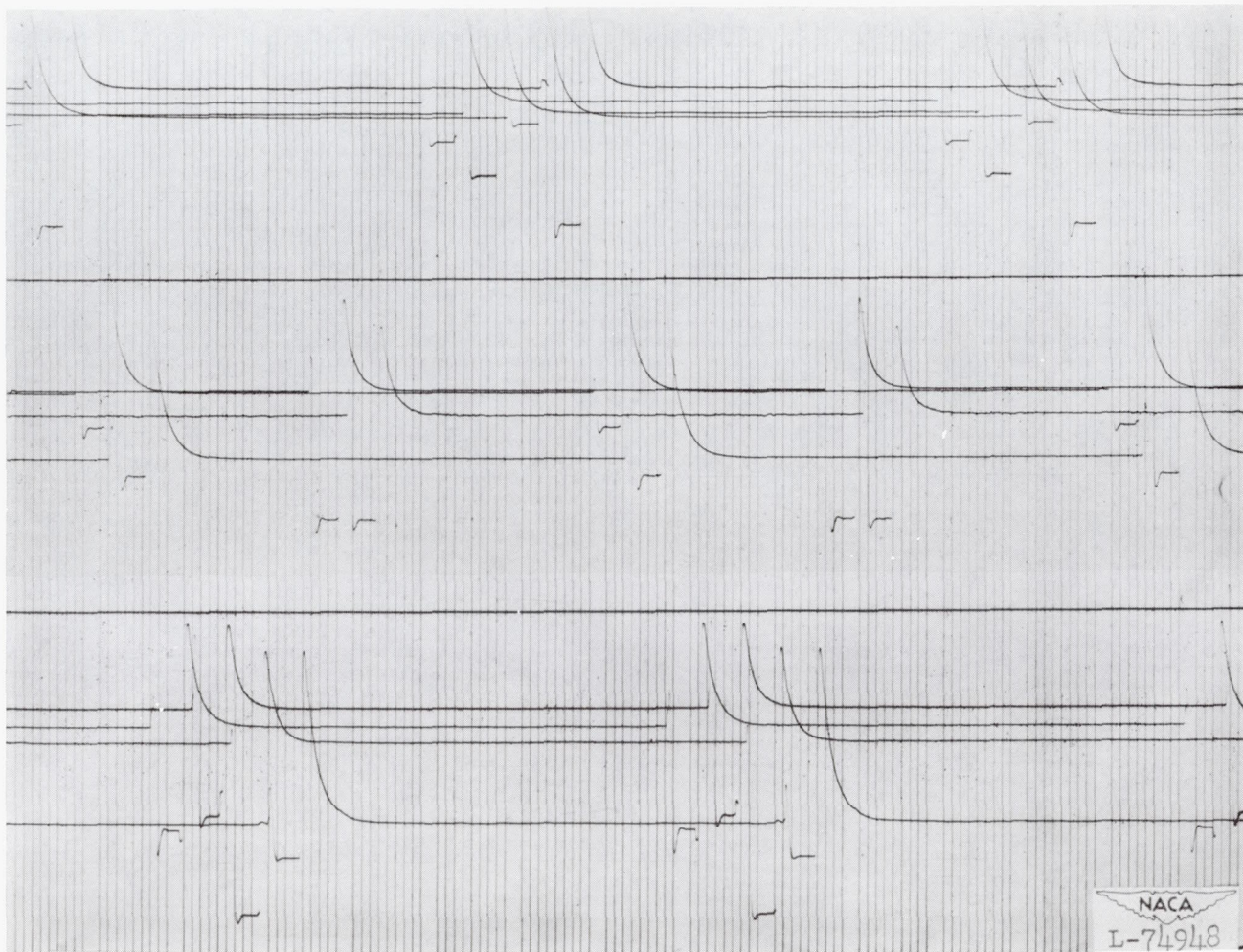


Figure 6.- Sample record of surgical-lamp Pirani gage 12-channel low-pressure recorder.

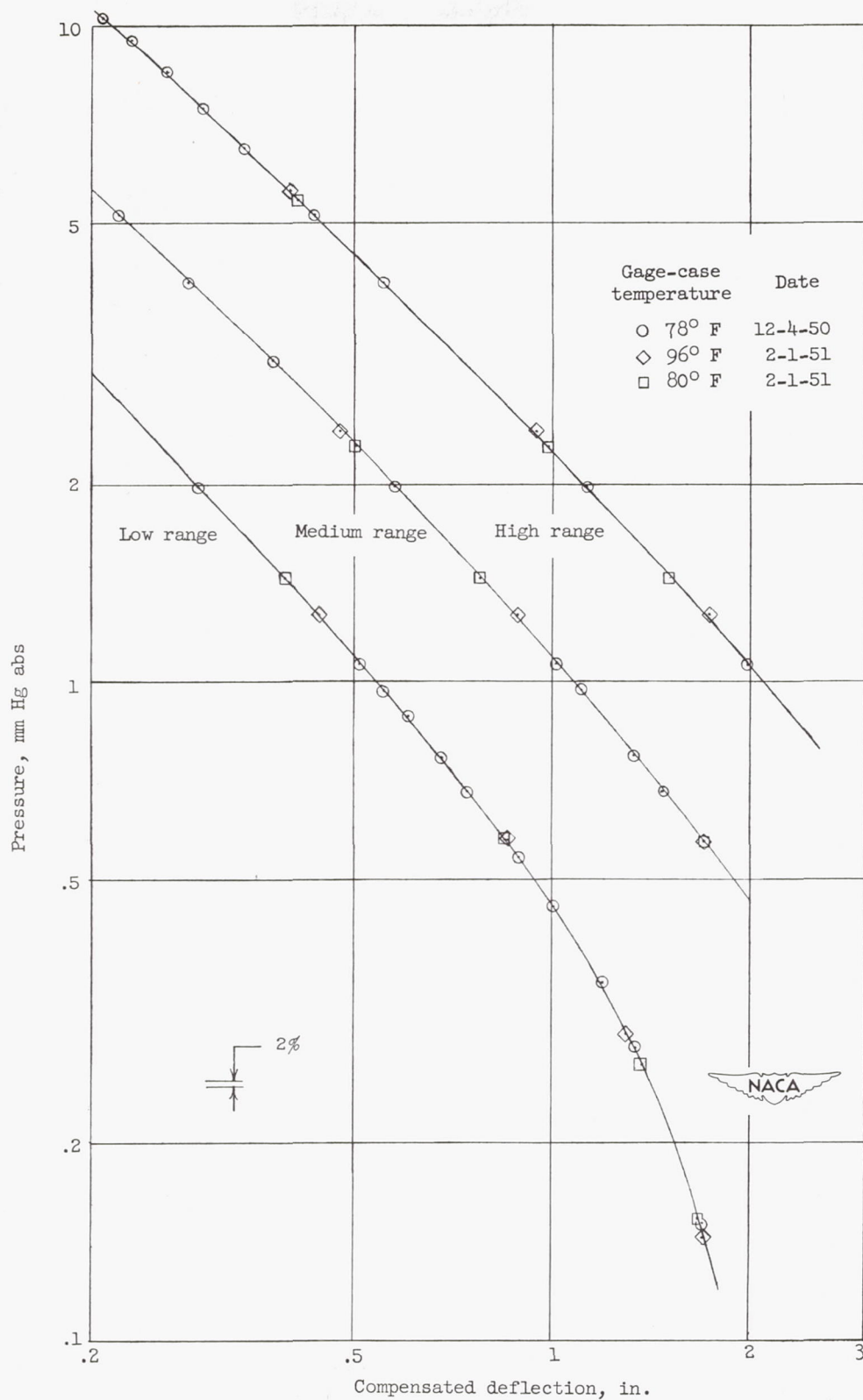


Figure 7.- Surgical-lamp Pirani gage sample calibration, approximately one-quarter scale.

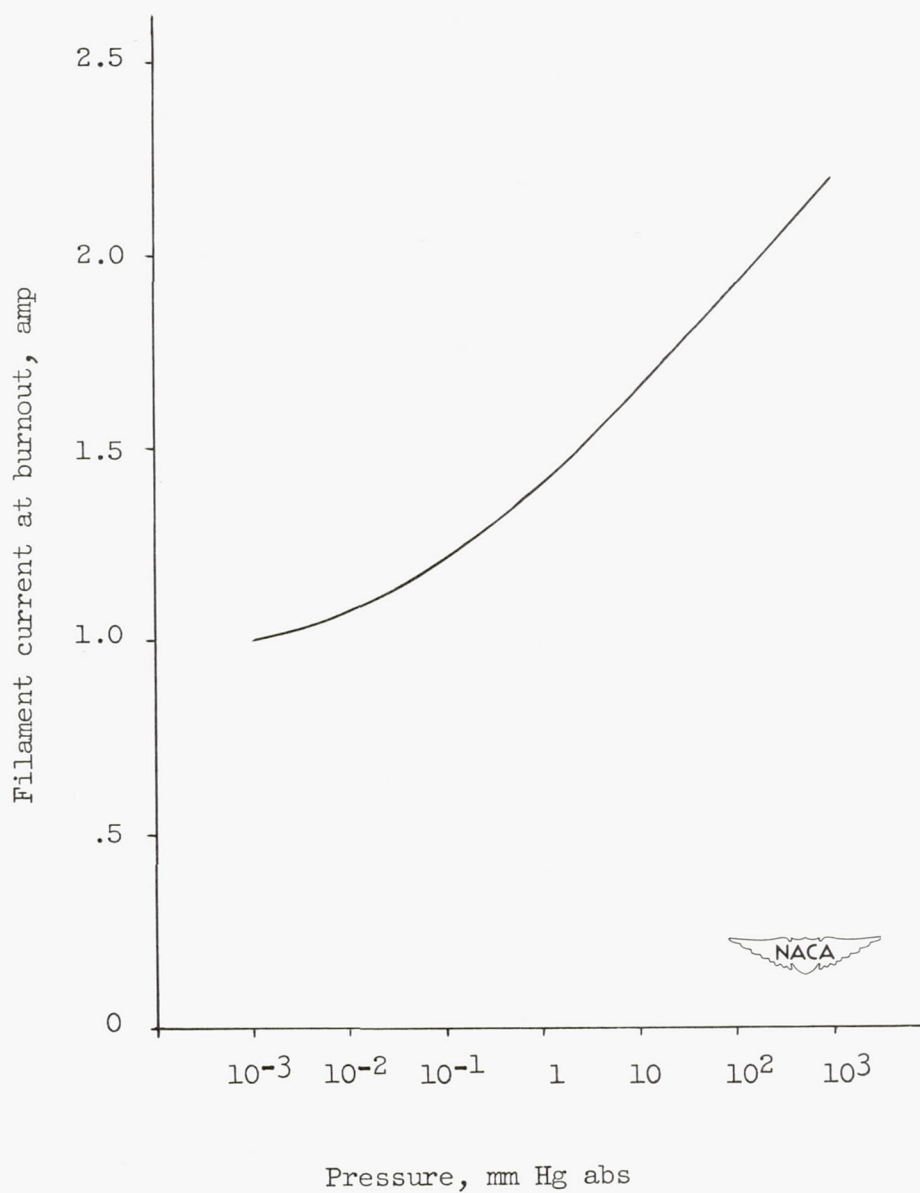


Figure 8.- Average burnout characteristic curve of open surgical grain-of-wheat lamps.

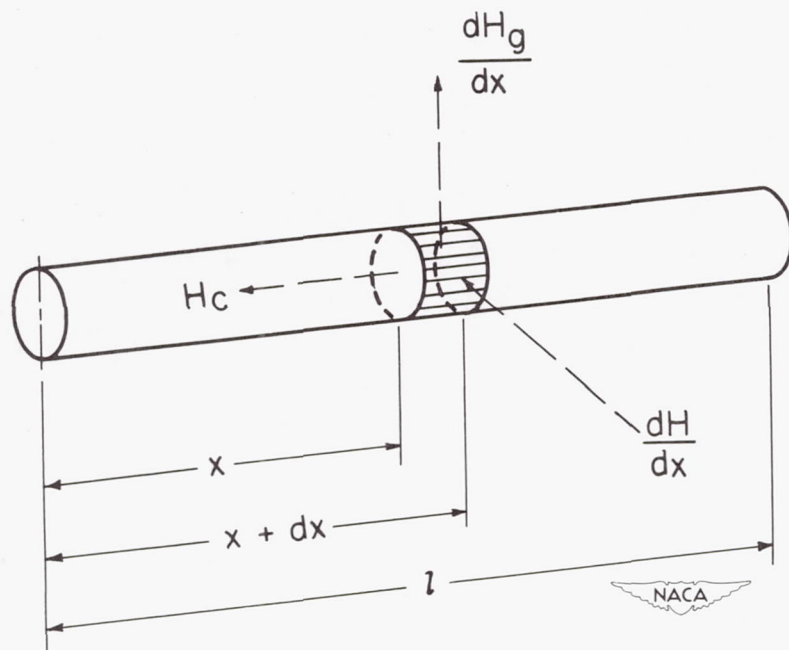


Figure 9.- Sketch showing combined effect of heat generated in filament and heat losses through filament and through gas.

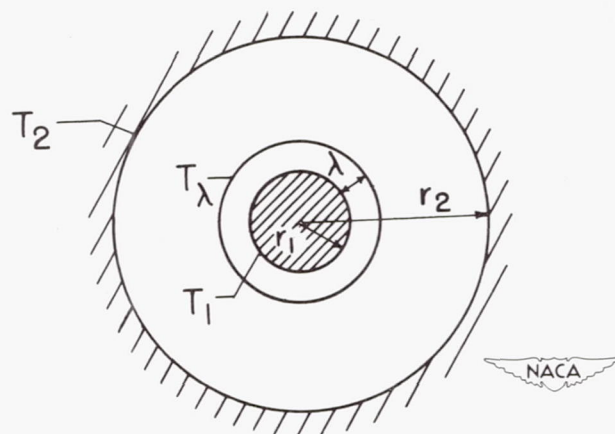


Figure 10.- Sketch showing heat conduction through gas from filament.

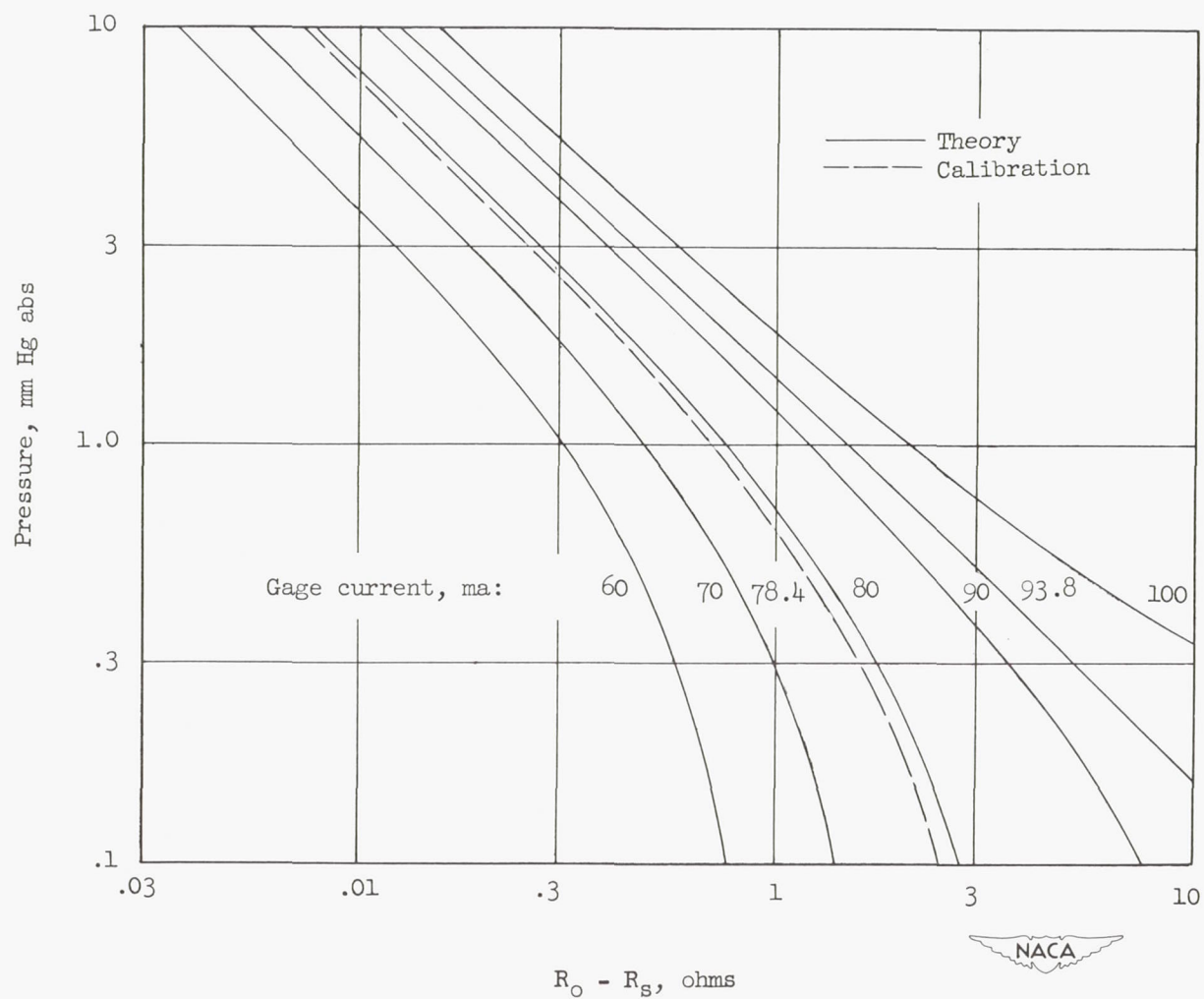


Figure 11.- Theoretical and measured characteristics of a small Pirani gage made of surgical grain-of-wheat lamps measuring pressure of dry air at 80° F.